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REVIEW ARTICLE

Recent experimental studies of electron transport in open quantum dots

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Abstract. Recent advances in semiconductor microprocessing technology now allow the realization of sub-micron sized *quantum dots*, which are quasi-zero-dimensional devices in which current flow is confined on length scales approaching the Fermi wavelength of the electrons. The influence of disorder is thought to be strongly suppressed in these devices, so that electrons propagate while mainly undergoing large-angle scattering at the walls of the dot. At sufficiently low temperatures, electron phase coherence is maintained over long distances and coherent interference of electrons becomes an important process in determining the electrical behaviour of the dots. In this review, we focus on a number of issues revealed by recent experimental studies of open dots, such as fractal magneto-conductance fluctuations, wavefunction scarring due to selectively excited periodic orbits, and novel 'ratchet' behaviour in non-equilibrium studies.

1. Introduction

Recently, much interest has focused on the use of semiconductor *quantum dots* as a novel experimental probe of quantum chaos, which is a rapidly expanding field of research concerned with the nature of the crossover between the classical and quantum regimes [1–3]. Quantum dots themselves are quasi-zero-dimensional semiconductor structures in which the flow of electrical current is confined on length scales approaching the Fermi wavelength of the electrons. At sufficiently low temperatures, phase coherence of the electron wavefunction is maintained over long distances and the transport properties of the dots are determined by a variety of quantum mechanical phenomena, including single-electron tunnelling [4,5], discrete energy quantization [6, 7], coherent electron wave interference [8–10], and spin-regulated transport [11]. From the perspective of quantum chaos, interest in the properties of quantum dots derives from the ability to fabricate devices in which the influence of disorder scattering is strongly suppressed. In such *ballistic* dots, large-angle scattering of electrons occurs mainly at the confining walls of the dot, whose geometry is therefore expected to play a crucial role in determining the overall device behaviour.

The key components of a quantum dot are shown in figure 1 and consist of a sub-micron sized central *cavity* that is connected to external reservoirs by means of waveguide *leads*. In most situations we consider here, the dots are formed using the split-gate technique [12], the details of which are discussed in the following section. The use of electron-beam lithography to define the split-gate structures allows the realization of dots on the sub-micron scale, which size is comparable to the spatial extent of the potential fluctuations that exist in the underlying electron gas [13, 14]. A particular advantage of the split-gate approach is that the dot leads may be realized using *quantum point contacts*, whose width can be varied continuously in

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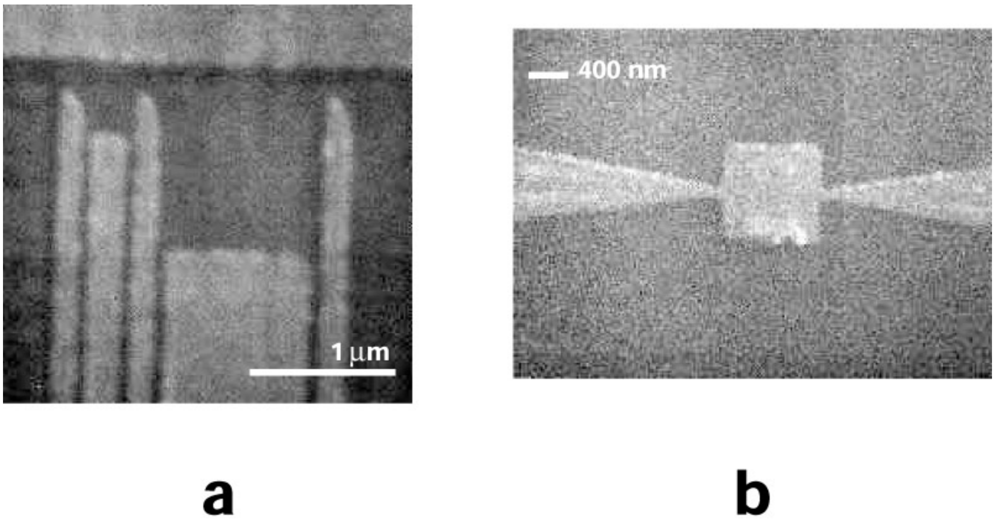


Figure 1. Scanning electron micrographs showing the different gate structures that may be utilized to realize quantum dots. (a) Split-gate dot structure in which the application of a negative bias to the metal gates (lighter regions) results in the formation of a coupled quantum dot. Picture courtesy of A Adresen and C Prasad. (b) In this structure the metal gate acts as a self-aligning mask and protects the underlying electron gas during an etching step. Variation of the bias applied to the gate therefore allows the electron number in the dot to be varied. Picture courtesy of D P Pivin Jr.

experiment. With the leads configured to form tunnel barriers, current flow through the dot can only occur by tunnelling, the details of which are known to be determined by the Coulomb blockade effect [4]. In this review, however, we choose to focus on the behaviour exhibited by *open* quantum dots, whose leads are configured to support one or more propagating *modes*. The Coulomb blockade effect is thought to be suppressed in such open dots [5, 15] (we briefly return to a discussion of this issue in section 7), in which electron transport instead provides a natural connection to the study of quantum chaos.

Initial studies of the transport properties of open dots were performed by Marcus *et al* who measured the magneto-resistance of circular- and stadium-shaped dots [16]. At temperatures below a degree kelvin, the magneto-resistance of the dots was found to exhibit reproducible fluctuations, which were ascribed to coherent interference of geometrically scattered electrons [17] (figure 2). The spectral content of the fluctuations was found to be different for the two dot geometries studied and this behaviour was argued to result from the influence of the classical scattering dynamics on the quantum transport behaviour (the circle is expected to give rise to regular scattering while the stadium is known to be *chaotic*). Another feature of this experiment, that was confirmed in subsequent studies [18–20], was the observation of a zero-field peak in the magneto-resistance, which was later ascribed to the ballistic analogue of weak localization [21]. Based on semi-classical arguments, it was proposed that the lineshape of this peak should provide a probe of electron scattering in the dots, with a Lorentzian form predicted for chaotic scattering and a linear lineshape expected for regular dynamics [21]. These predictions appeared to be convincingly confirmed in a later experiment by Chang *et al* who studied the magneto-resistance of multiply connected arrays of circular- and stadium-shaped dots [22].

Subsequent to the studies described above, a large number of experiments have been performed to investigate different aspects of electron dynamics in open dots. The purpose of this review is to provide a general overview of these studies, which is organized as follows.

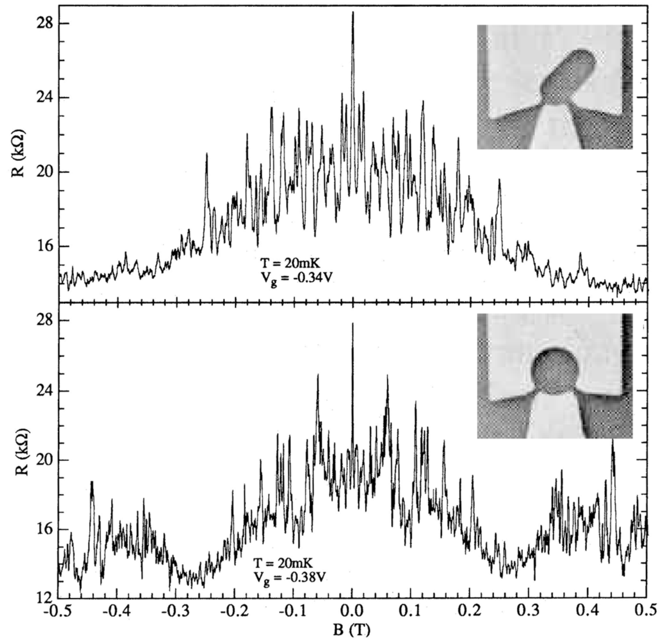


Figure 2. The magneto-resistance of stadium- (top) and circular-shaped dots is found to exhibit pronounced and reproducible fluctuations at low temperatures. Figure reproduced with permission from [16]. Copyright 1992 by the American Physical Society.

In section 2, we consider some of the techniques available for the fabrication of quantum dots, and focus, in particular, on the use of gated nanostructures. Since an important parameter for determining the influence of quantum effects in mesoscopic devices is provided by the electron phase-breaking time, in section 3 we summarize the results of experiments that have been performed to measure this parameter directly. In section 4, we discuss the results of recent experiments that demonstrate the *fractal* nature of the magneto-conductance fluctuations in open dots, while in section 5 we consider the important role played by selectively excited orbits in the dots. At suitably low temperatures, interference of these orbits gives rise to marked wavefunction scarring and in section 5 we consider the experimental signatures of this scarring. An alternative interpretation of the zero-field magneto-resistance peak, which ascribes its origin to energy averaging of the discrete level spectrum in the dots, is next discussed in section 6. In section 7, we consider some outstanding issues raised by the studies performed to date, before concluding in section 8. Since we focus on the behaviour exhibited by *open* dots, the reader should note that one issue we do not consider is the use of *weakly coupled* dots to investigate the predictions of random matrix theory [23–26]. This topic has recently been well served by the thorough review of Beenakker [27].

2. Experimental techniques for quantum dot realization

While a number of different techniques are available for the fabrication of quantum dots [28], the split-gate technique [12] is the preferred choice in studies of quantum chaos. In this approach, metal gates with a fine-line pattern that is defined by electron beam lithography are deposited on the surface of a GaAs/AlGaAs heterojunction. Application of a suitable negative bias to the gates depletes the regions of electron gas from directly underneath them, forming a dot whose lead openings are defined by means of quantum point contacts [29, 30] (figure 1(a)). The use of electron beam lithography to pattern the gates allows the realization of sub-micron sized dots, whose size is therefore comparable to the spatial extent of the short-range potential fluctuations in the underlying two-dimensional electron gas [13]. These fluctuations are associated with the statistical distribution of donors in the AlGaAs layer and recent studies suggest that minimum-energy considerations cause the donor ionization to order into a quasi-regular lattice [14]. In the presence of the resulting weak disorder, electronic motion within the dots should then be predominantly ballistic in nature, with large-angle scattering events being largely restricted to the confining walls [31]. Another advantage of the split-gate approach is that, by the definition of a *multi-gate* dot structure such as that of figure 1(a), it is possible to perform transport ensemble averages *within the same dot* [32]. This approach has previously been exploited to compare the averaged transport properties of open dots to the predictions of random matrix theory [23–26].

In a variation on the split-gate technique, a single gate can be deposited on the semiconductor surface and used as a self-aligning mask to protect the underlying electron gas during a subsequent etching (or exposure) step [18, 33, 34] (figure 1(b)). A particular advantage of this approach is that, after definition of the cavity by the etching step, the metal gate may be biased to modulate the effective carrier density in the dot. Keller *et al* have fabricated such devices, for example, by exposing the ungated regions of the sample to a low energy dose of Xe ions [18], while Lee *et al* have used argon ions of similar energy [33]. Alternatively, Pivin *et al* have realized such dots (figure 1(b)) by exposing the metallized sample to a reactive ion etch [34].

3. Studies of electron dephasing in quantum dots

One of the most important parameters for determining the strength of quantum effects in mesoscopic devices is the phase-breaking time (τ_ϕ), which is essentially the average time over which the wavelike nature of the electrons is preserved [28]. The study of phase-breaking has a long history in disordered systems in which, at the low temperatures of interest here, the main source of dephasing is thought to be due to electron–electron scattering. Our understanding of dephasing in ballistic quantum dots, on the other hand, remains at a primitive level and is based almost exclusively on the results of a small number of experiments [32, 34–38]. Theoretical calculations of the temperature dependence of the phase-breaking time are even fewer in number, although Sivan *et al* have considered the dephasing rate due to electron–electron scattering in an *isolated* dot [39]. These authors obtain a phase-breaking time that varies with temperature (T) as $\tau_\phi \propto T^{-2}$, although it is not clear whether their model is relevant to the *open* dots of interest here. More recently, however, Takane [40] considered phase-breaking due to Coulomb interactions in a chaotic dot and found that $\tau_\phi \propto T^{-1}$. This is essentially similar to the behaviour found for electron–electron scattering in two-dimensional systems and, as we will discuss below, is also much closer to the temperature dependent behaviour found in a number of different experiments.

In experiment, Marcus and co-workers have developed a number of techniques for extracting phase-breaking times that derive from the assumption of chaotic electron scattering in the dots [32, 35, 41]. In one such approach, the change in the spectral content of the magneto-conductance fluctuations in small dots is measured as the width of their lead openings is varied. To obtain an estimate for the phase-breaking time, the effective escape rate of *coherent* electrons from a chaotic dot is exploited [35, 41]:

$$\gamma_{eff} = \frac{1}{\tau_\phi} + \gamma_{esc} \tag{1}$$

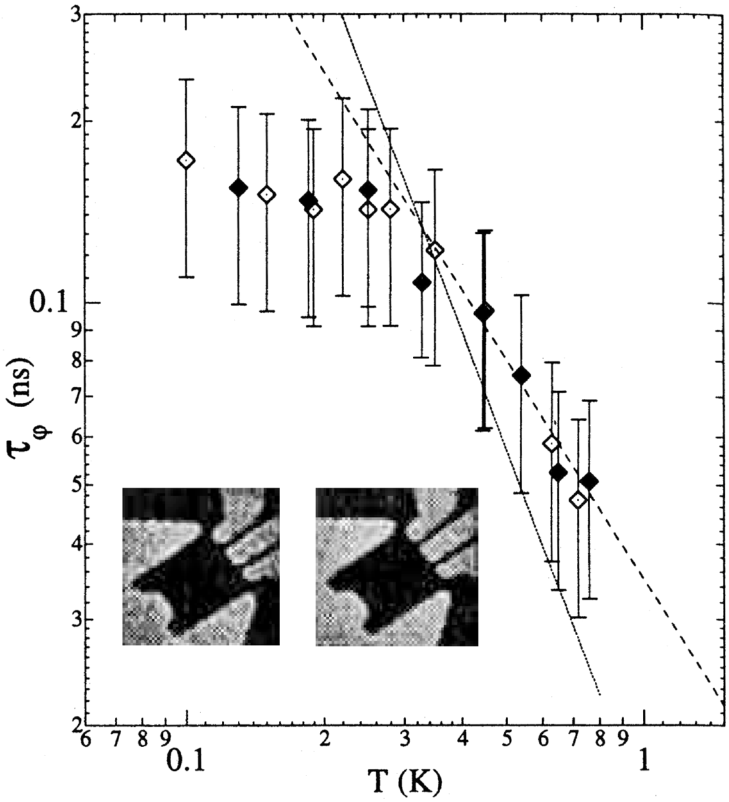


Figure 3. The variation of phase-breaking time with temperature measured by Clarke *et al* . Solid and open symbols are the results for two different dots. The dotted line shows the T^{-2} variation predicted for isolated quantum dots, while the dashed line indicates a $T^{-1.2}$ dependence. SEM micrographs of the two dots studied are shown as insets, in which the lighter regions correspond to the split-gate structure. The inner dimension of both dots is roughly 2 μm. Figure reproduced with permission from [35]. Copyright 1994 by the American Physical Society.

where γ_{esc} is the rate at which electrons leave the dot via either of its leads. Since γ_{eff} determines the spectral content of the conductance fluctuations [35, 41], using equation (1) it is possible to extract an estimate for τ_ϕ from these fluctuations. (An important assumption here is that the dephasing rate should be *independent* of the width of the lead openings.) Using this approach, Clarke *et al* [35] obtained estimates for the phase-breaking time and the results of their analysis are summarized in figure 3. Around a degree kelvin, the data show something close to a T^{-1} variation, while a saturation is observed at much lower temperatures. Here, the phase-breaking time is of order a few hundred picoseconds, which is comparable to the values obtained in other mesoscopic systems [42, 43]. In a later experiment by Huibers *et al* [32], the amplitude of the ensemble-averaged zero-field peak was compared to the predictions of random matrix theory [27, 44] in order to obtain estimates for τ_ϕ. Similar to the study of Clarke *et al* these authors found low temperature values for τ_ϕ of the order a few hundred picoseconds and a variation with temperature close to T^{-1}. One important difference, however, was that *no* evidence for a low temperature saturation was seen in their experiment, suggesting that the saturation seen previously may not be intrinsic to the dots. Even more surprisingly, these authors were able to fit their data for the temperature dependence of τ_ϕ

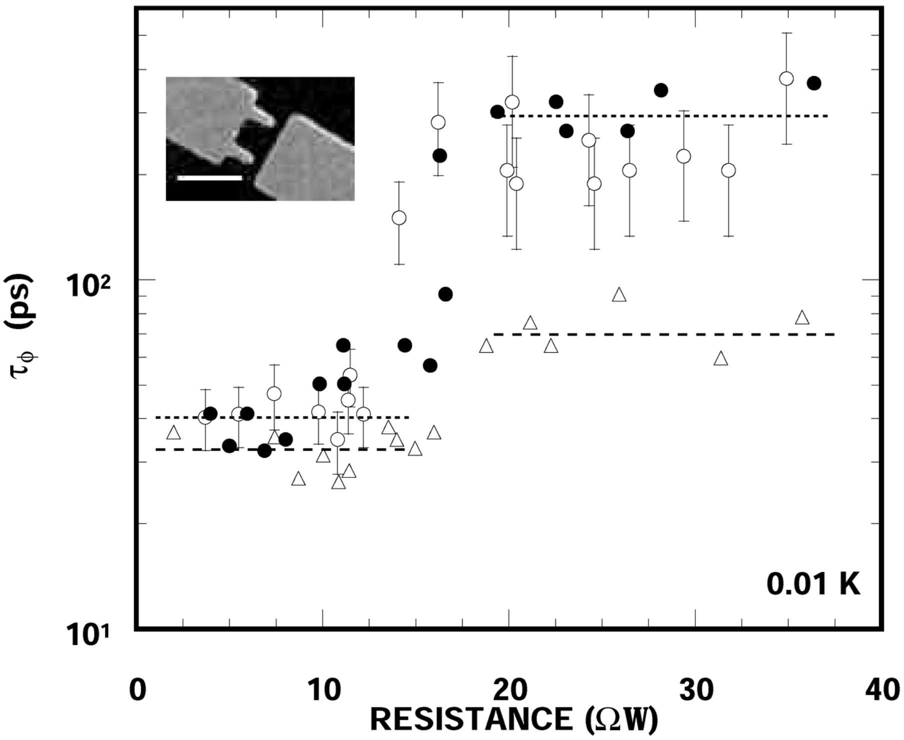


Figure 4. Variation of the phase-breaking time with lead opening, measured in three different dots with nominally square gate geometry. The solid circles are for a 1 μm dot, the open circles are for a 0.4 μm dot and the open triangles are for a 0.6 μm dot. Lines are intended to guide the eye and additional error bars are omitted for clarity. The inset shows an SEM micrograph of a 0.6 μm dot in which the spacer bar represents 1 μm. Figure reproduced with permission from [38].

using theoretical predictions for two-dimensional *disordered* systems, although the success of this approach relied on the introduction of a fitting parameter whose significance is still not well understood.

In an alternative approach to those above, Bird *et al* [36, 38] have determined values for the phase-breaking time using a model that was originally developed to describe the properties of the magneto-conductance fluctuations in quasi-ballistic quantum wires [45]. This approach does not require the assumption of chaotic scattering in the dot but instead determines τ_ϕ from the magnetic field dependence of the conductance fluctuations in the edge state [46, 47] regime. In temperature dependent studies, these authors have found that their data for τ_ϕ are best fitted by a T^{-1} variation, consistent with the experiments above. They also find a saturation of τ_ϕ at temperatures lower than the average level spacing in the dot (Δ, where $\Delta = 2\pi\hbar^2/m^*A$ and A is the effective area of the dot), a conclusion that is supported by the scaling of the saturation temperature with dot size [48]. Bird *et al* have also used the same technique to determine the variation of the phase-breaking time with lead opening and find a strong enhancement of τ_ϕ as the leads are narrowed to support a few modes [38] (figure 4). While the origin of this enhancement remains undetermined, the authors have suggested that a suppression of the phase space available for electron scattering, which occurs as the dot leads are narrowed, may be responsible. Further emphasizing the importance of the lead openings in quantum dot transport, Pivin *et al* [34] recently studied phase-breaking in a quantum dot

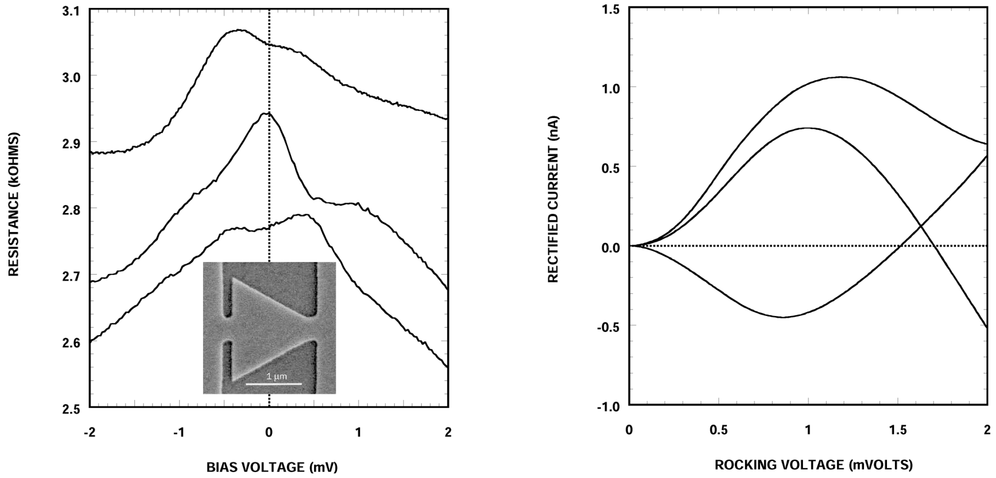


Figure 5. Left figure: low-temperature (0.3 K) measurements of the differential resistance versus bias voltage for the triangular dot shown in the inset. The different curves were obtained by varying the Fermi energy in the dot using an appropriate top gate structure. Right figure: net rectified current obtained in the asymmetric billiard when an ac 'rocking' bias voltage is applied. Note how the sign of the rectified current depends on the amplitude of the rocking voltage. For a fuller discussion of this phenomena see [52]. Figure provided courtesy of H Linke.

coupled to long *quantum wires* (figure 1(b)). These authors found a saturation of τ_ϕ that persists to temperatures much higher than the average level spacing, followed by a high-temperature decay that is proportional to $T^{-2/3}$. Since this latter variation is precisely that expected for dephasing in a one-dimensional wire, the authors suggest that the high-temperature decay of the phase-breaking time may be related to the properties of the *reservoirs* to which the quantum dot is coupled. The origin of their low-temperature saturation remains undetermined, though.

A valuable probe of phase coherence in quantum dots should be provided by non-equilibrium transport studies, although relatively few such reports have appeared to date [49–52]. Linke *et al* [49, 50] used the properties of the zero-field resistance peak to compare the dephasing of equilibrium and non-equilibrium electrons and found a common dependence of τ_ϕ on energy. At higher energies, the phase-breaking time was found to vary as an inverse square law of temperature, consistent with the theoretical predictions of Sivan *et al* [39]. At energies less than or comparable to the average level spacing, however, the phase-breaking time was found to saturate, similar to the behaviour discussed above [36, 48]. Switkes *et al* [51] studied the influence of measurement current on electron temperature and found this to be determined by an equilibrium condition where the energy supplied to the dot by non-equilibrium electrons is balanced by loss to the reservoirs via the point contact leads. Probably the most exciting issue revealed by these studies, however, is the observation of 'ratchet' behaviour in asymmetric triangular billiards [52] (figure 5). A crucial property of such billiards is their lack of spatial inversion symmetry, which may give rise to a net drift of electrons even when the time-average of externally applied forces is zero. Linke *et al* [52] measured the differential resistance of a triangular billiard as a function of the dc source–drain bias and found this to exhibit marked asymmetries on reversal of the bias direction (figure 5). These in turn were attributed to asymmetries that are generated in the confining profile of the dot on reversal of the dc bias. Remarkably, these authors found that the asymmetric geometry of the dots can actually cause *rectification* of an ac current. The rectification vanishes at temperatures above a few kelvin, indicating that its origin lies in a quantum interference effect. This should be contrasted with the rectification effect recently reported by Song *et al* [53], who studied the properties of ballistic junctions incorporating asymmetric scattering centres and found their I–V characteristics to be highly asymmetric. This latter behaviour was found to persist (albeit weakly) to liquid nitrogen temperatures and can essentially be accounted for in terms of a classical ballistic effect.

4. Fractal magneto-conductance fluctuations in open dots

Original theoretical studies of electron scattering in quantum dots employed a semi-classical approximation in which electrons were assumed to follow classical trajectories while accumulating a quantum mechanical phase [17, 21]. According to this approach, the dot conductance could then be computed by exploiting the fact that, for particles trapped in a *hard-walled* dot, the number of trajectories enclosing a specific area is given as [54–56]:

$$\begin{aligned} N_{chaotic}(A) &= N_0\,\mathrm{e}^{-\alpha A} \\ N_{regular}(A) &= N_0 A^{-\gamma}. \end{aligned} \tag{2}$$

Here, A is the enclosed trajectory area, α and γ are suitable constants, and $N_{chaotic}$ and $N_{regular}$ are the area distribution functions for chaotic and regular billiards, respectively.

A problem with the arguments above is that the confining profiles of quantum dots are typically *soft walled* in nature [14] (this is particularly true for the split-gate dots of interest here). Ketzmerick pointed out that the effect of such potentials is to generate a *mixed* phase-space for electron motion, consisting of a sea of chaotic trajectories that is interspersed with an infinite hierarchy of stable cantori [57] (figure 6(a)). In such mixed systems, the area distribution typically takes a *power-law* form, which arises when chaotic trajectories become *trapped* in the vicinity of regular orbits (figure 6(b)), and the resulting conductance fluctuations are expected to be *fractal* in nature. That is, the fluctuations are expected to exhibit structure on a number of different magnetic field scales (figure 6(a)). When the area distribution in the dot takes the form $N(A) \propto A^{-\gamma}$, Ketzmerick was able to show that the fractal dimension (D_F) of the conductance fluctuations should be given as:

$$D_F = 2 - \frac{\gamma}{2}. \tag{3}$$

For fractal fluctuations, $1 < D_F < 2$ which in turn implies that $0 < \gamma < 2$. The fractal dimension is easily determined from experimental magneto-conductance curves using standard approaches such as box-counting. Here, the number (N) of non-overlapping boxes of area ΔB^2, required to completely cover a magneto-resistance trace, is counted as a function of the box-width (ΔB). The fractal dimension is then given as [58]:

$$D_F = \lim_{\Delta B \to 0} -\frac{\log N\Delta B}{\log \Delta B}. \tag{4}$$

One of the first experiments to demonstrate the fractal nature of fluctuations was performed by Hegger *et al* [59], who studied the low temperature magneto-conductance of gold nanowires and found $1.05 < D_F < 1.18$. Subsequently, fractal fluctuations have been reported in a number of different studies of quantum dots [60–63]. Micolich *et al* [60] studied the fluctuations in a square-shaped dot and, at the lowest experimental temperatures, found values for D_F as high as 1.38. These authors also studied how the fractal dimension is modified at higher temperatures, where phase-breaking imposes an upper limit on the trajectory length that may be traversed coherently by electrons. Rather than suppressing the fractal fluctuations, increased dephasing was found to lead to a corresponding decrease in the fractal dimension. Indeed, in a later report, the same authors were able to demonstrate a direct correlation between

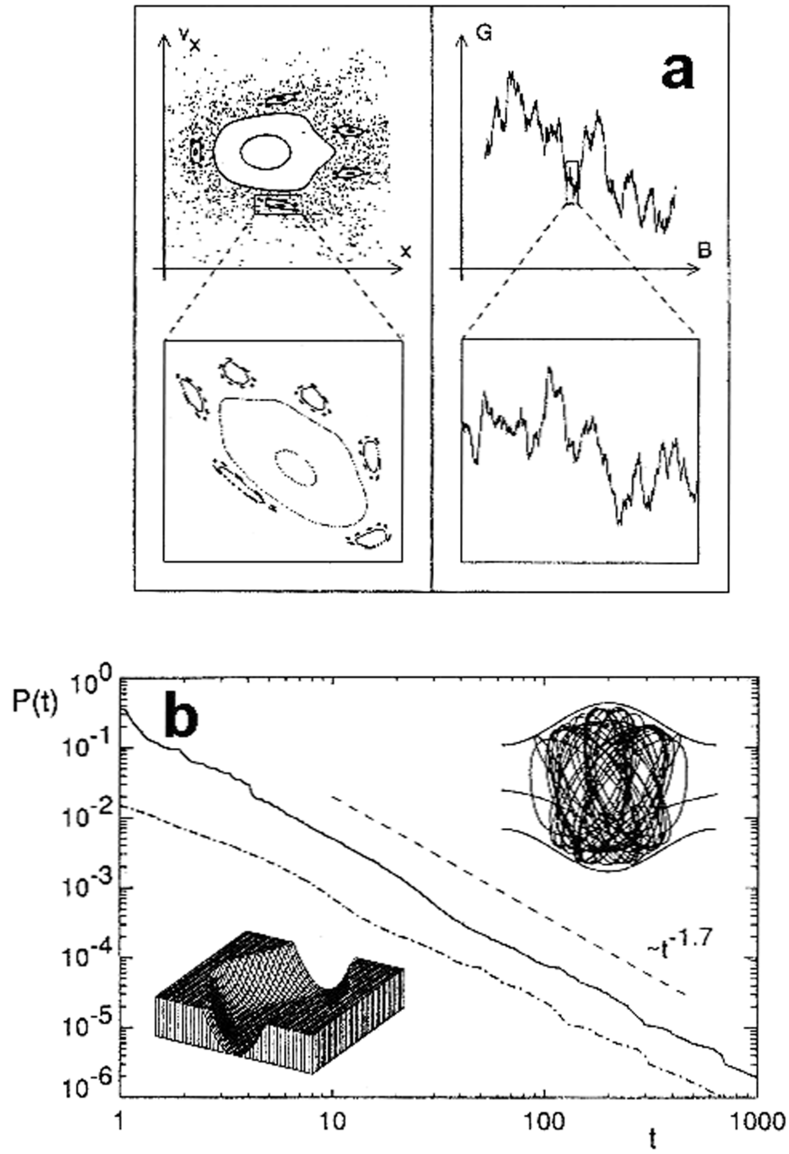


Figure 6. (a) Hierarchical phase space structure of a 2D chaotic system and associated fractal conductance fluctuations. (b) The probability ($P(t)$) of staying trapped in a cavity for a time greater than t. Note the *power-law* decay of this probability. The upper right inset shows a typical trajectory trapped in the hierarchical phase space structure giving rise to the power-law behaviour. The lower inset shows the soft-walled cavity structure employed in the numerical simulations. Figure reproduced with permission from [57]. Copyright 1996 by the American Physical Society.

the fractal dimension and the phase-breaking time [63] (figure 7). Variation of the split-gate voltage was also found to modify the fractal dimension and was attributed to the influence of the gate voltage on the soft profile of the dot. Here it was suggested that, by modifying the detailed profile of the dot, the effect of the gate voltage is to alter the trajectory-trapping effect mentioned above. The influence of gate voltage was studied in greater detail by Sachrajda

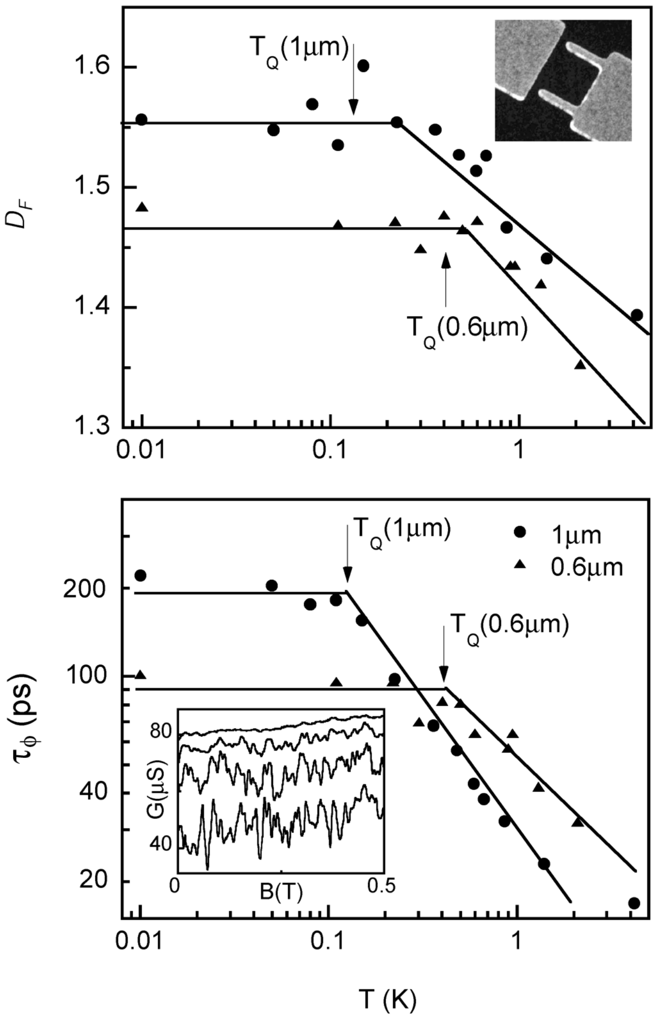


Figure 7. A comparison of the variation of the fractal dimension (upper plot) and phase-braking time (lower plot) with temperature for two different sized quantum dots. The 1 μm dot geometry studied is shown as an inset to the upper plot. The inset to the lower plot shows measured convectance fluctuations in the 1 μm dot at temperatures of 4.2 K, 1.4 K, 0.48 K and 0.08 K, from top to bottom, respectively (the curves are offset for clarity). Figure provided courtesy of A P Micolich.

et al [61], who found that the gate-voltage could be used to change the fractal dimension *continuously* in the range $1.05 < D_F < 1.4$, with D_F tending towards unity as the strength of the confining potential was reduced (figure 8).

The fractal behaviour discussed above provides a good example of *statistical* self-similarity [57]. Recently, however, Taylor *et al* provided remarkable evidence for the presence of *exactly* self-similar structure in the magneto-conductance of a Sinai billiard [64]. This billiard was defined by fabricating a bilayer gate structure in which the lower set of gates was used to define a square dot while the upper gate was used to form a tunable anti-dot at the centre of the billiard (figures 8 and 9). For particular values of the voltages on these gates, it was found that the magneto-resistance exhibited exact self-similarity, according to which the resistance

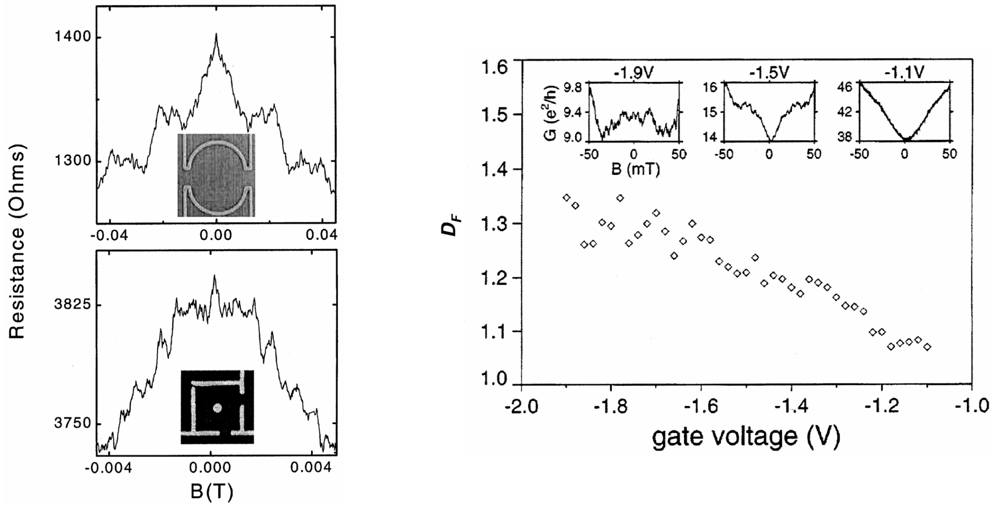


Figure 8. Measurement of fractal conductance fluctuations in two different dot geometries. The upper left figure shows the low-temperature (0.05 K) fluctuations measured in a stadium-shaped dot while the lower left figure shows corresponding measurements for a Sinai dot geometry. The right figure shows the variation of the fractal dimension of the stadium-shaped dot with gate voltage. Measurements of the fluctuations in this dot at different gate voltages are shown in the inset to this figure. Figure reproduced with permission from [61]. Copyright 1998 by the American Physical Society.

looked the same when viewed at different magnifications (figure 9). Such exact self-similarity was not anticipated in the original work of Ketzmerick and since its observation a number of attempts have been made to account for its origin [65–67]. While none of these studies have been able to clearly reproduce the behaviour seen in experiment, it nonetheless seems clear that the exact form of the self-similarity implies a highly selective nature of electron transport in the dot. Fromhold *et al* [65] have suggested that the coarse structure observed in the magneto-resistance is related to orbits that enclose the central anti-dot, while the fine and ultra-fine structure is related to much longer semi-classical paths. The difference between statistical and exact self-similarity may be quantified by defining the magnetic field and conductance *scaling factors* λ_B and λ_G, respectively [62, 68]. These factors connect the structure observed at different magnetic field and conductance scales and, in the case of exact self-similarity, take only a *limited* set of values. In contrast, for statistically self-similar fluctuations, there exists a *continuous* range of scaling factors [68].

In a study of the fluctuations in a stadium and a Sinai dot geometry, Sachrajda *et al* noted that the exact self-similarity of the magneto-conductance appears to be a characteristic of the Sinai geometry (figures 8 and 9) alone [61]. Support for this idea is provided by the later work of Taylor *et al* who studied the magneto-conductance in three different dot geometries [62] ((a)–(c), figure 10). These authors found that the fluctuations measured in *empty* dots (a) and (b) typically exhibit *statistical* self-similarity, suggesting that the observation of exact self-similarity in geometry (c) results from the presence of the anti-dot *inside* the billiard geometry. To allow the transition from statistical to exact self-similarity, it was suggested that the presence of this anti-dot gives rise to an orbit selection process that does not occur in empty geometries. The nature of the selection process is not presently understood, however.

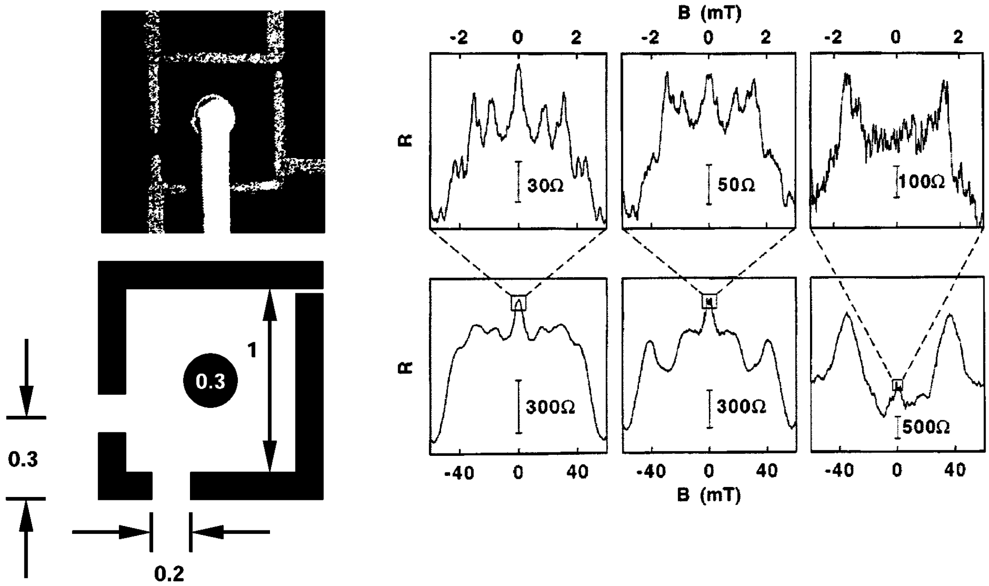


Figure 9. Evidence for exact self-similarity in the low-temperature (0.03 K) magneto-resistance of a Sinai dot geometry. The device geometry and its dimensions in microns are shown in the left hand side of the figure. The magneto-resistance curves shown were obtained by biasing both the outer gates and central anti-dot with a negative gate voltage. Figure reproduced with permission from [64]. Copyright 1997 by the American Physical Society.

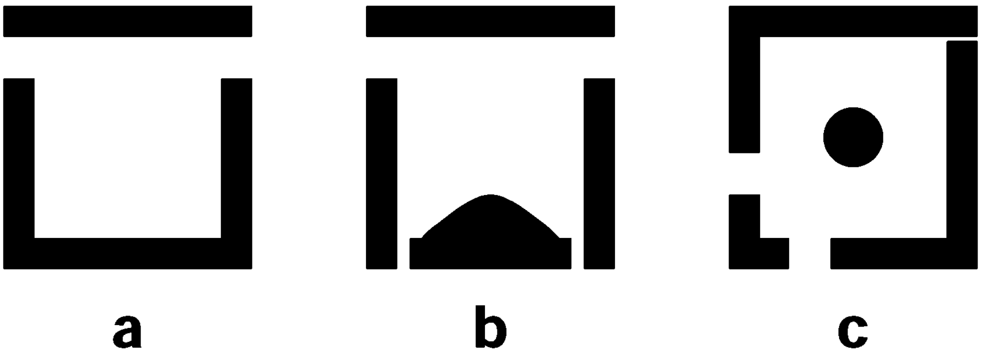
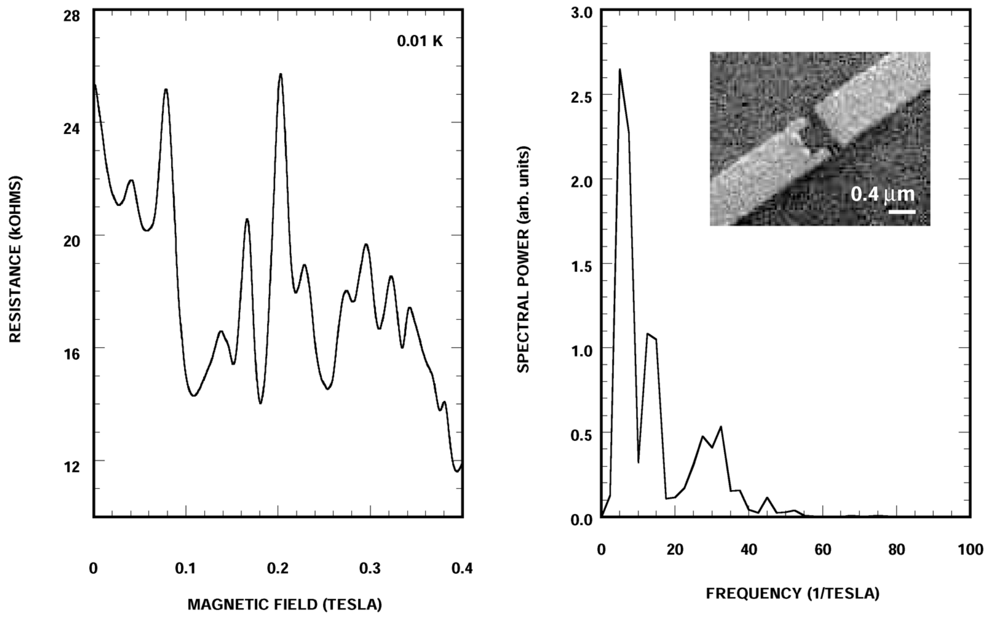


Figure 10. The different quantum dot geometries studied in [60]. (a) Square. (b) Empty Sinai billiard. (c) Sinai billiard with central anti-dot.

5. Periodic orbits in open quantum dots

Early studies [16] of the magneto-conductance fluctuations in open dots were motivated by the theoretical work of Jalabert *et al* who suggested that the spectral content of the fluctuations should sensitively depend on the type of electron dynamics in the dot [17]. The basic approach of these authors was to express the transmission probability of the dot as a *uniform* sum over all semi-classical trajectories connecting the input and output leads. In a dot that generates chaotic scattering, electrons are expected to sample phase space uniformly and the resulting



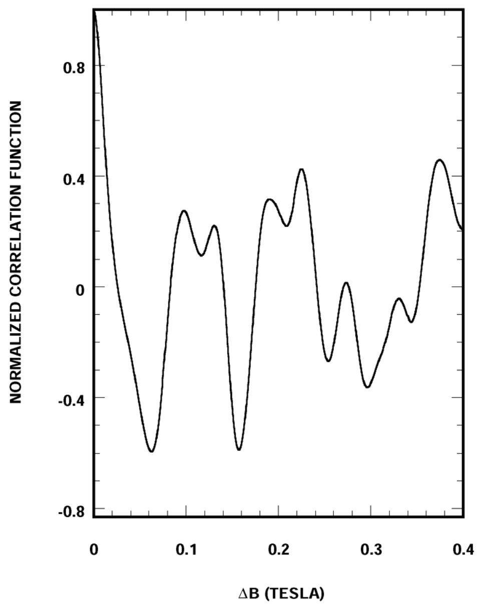


Figure 11. The low-temperature magneto-resistance of open quantum dots is found to exhibit highly reproducible fluctuations. Measurements were performed at 0.01 K on a 0.4 μm dot (for the dot geometry see the inset) and the raw fluctuations are shown in the left hand figure. A Fourier transform of these fluctuations is shown in the middle figure and is characterized by a few dominant frequencies. The correlation function of the fluctuations is shown in the right hand figure ad shows clear oscillations. In order to reduce statistical error in the tail, this correlation function was computed using a *fixed* number of data points to determine each value. Unpublished data.

transmission probability should then be determined by a *broad* distribution of electron paths. At temperatures where phase coherence is maintained over long distances, interference of electron partial waves that propagate along these paths is then expected to crucially influence the conductance of the dot. A magnetic field may be used to modulate this interference and is expected to give rise to highly *aperiodic* conductance fluctuations, by varying the magnetic flux that is enclosed between the different trajectories. Basically, the random (but *deterministic*) nature of these fluctuations may be understood to result from the *broad* distribution of electron trajectories that contribute to the interference [17].

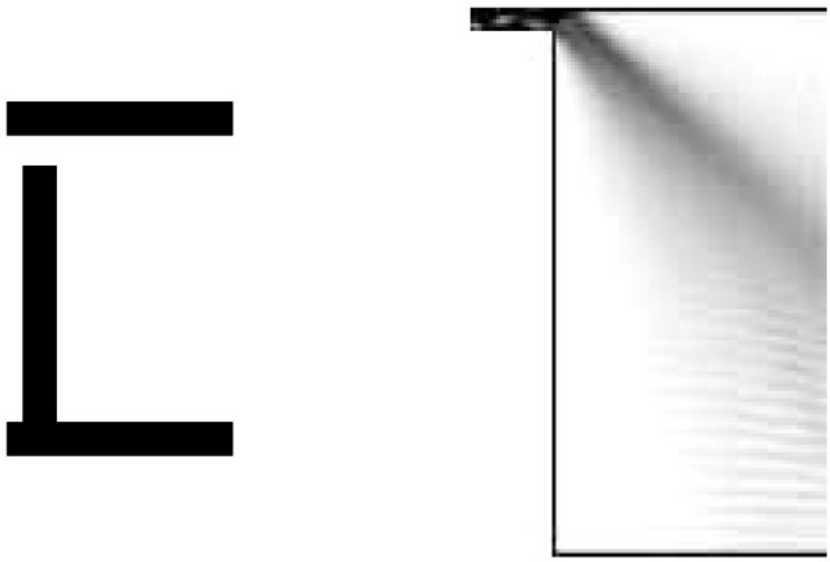
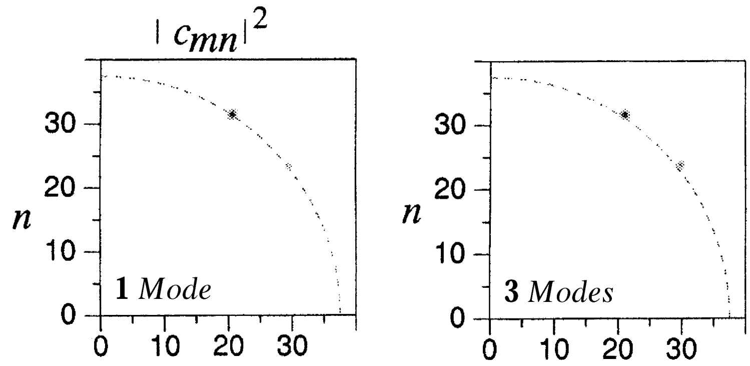


Figure 12. Numerical simulation of the electron probability density near a quantum point contact reveals significant *collimation* of the emerging beam. These simulations were performed using a lattice matching technique that is described in [75]. The left hand figure shows the particular geometry employed in the calculations while the squared amplitude of the electron wavefunction is plotted in the right hand figure. Darker regions correspond to higher probability density. Figure courtesy of R Akis.

The semi-classical model discussed above is only expected to be valid in situations where the point contact leads are configured to support a large number of modes. In most experiments, however, this condition is not usually satisfied and very different behaviour is typically observed. Rather than aperiodic fluctuations, the magneto-resistance of such few-mode dots instead shows quite regular oscillations [69–71] (figure 11). A Fourier analysis of the fluctuations reveals them to be dominated by a small number of frequency components, while their correlation function [72] shows a series of periodic oscillations in its tail. These oscillations imply the existence of long-range correlations in the data that are not expected for truly aperiodic fluctuations. Bird *et al* have studied the scaling properties of the fluctuations and have found their regular nature to be more pronounced in smaller dots, suggesting that a quantum size effect may at least be partially responsible for their observation [70, 71]. These authors also found that the dominant frequency components of the fluctuations are invariant to changes in either temperature or gate-voltage, a property that has been confirmed in the studies of Zozoulenko *et al* [73]. Motivated by these observations, Bird *et al* have suggested that electron transport in open dots is dominated by a small number of *selectively excited* and *highly stable* orbits. Interestingly, correlation functions showing similar oscillatory behaviour have also recently been reported in ion scattering studies of the $^{12}C + ^{24}Mg$ system [74], and have been attributed to the formation of stable states in the resulting hyperdeformed intermediate dinucleus. The observation of such common behaviour in very different physical systems is highly suggestive of a universal phenomenon and, moreover, points to significant limitations in the usefulness of random matrix theory when describing the behaviour of strongly quantized systems.

The regular nature of the fluctuations found in open dots may be accounted for by considering the properties of the point contact leads that couple the dots to their outside environment [75–78]. When configured to support a small number of modes, the quantum



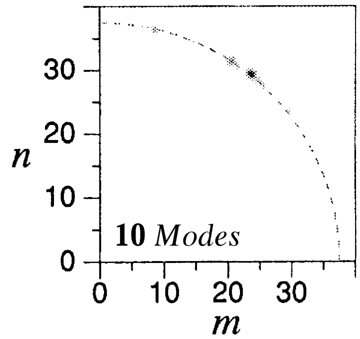


Figure 13. Expansion coefficients c_{mn} of the electron wavefunction in an open square dot, obtained for different numbers of occupied modes in the quantum point contact leads (indicated in figures). The expansion coefficients are computed by decomposing the electron wavefunction using equation (5) in the text. The dashed lines indicate the circle with radius $r = k_F L/\pi$, where L is the size of the dot. Figure reproduced with permission from [78]. Copyright 1997 by the American Physical Society.

mechanical nature of these leads is strongly resolved [75]. Consequently, electrons incident from the external reservoirs are only able to enter the dot by matching their transverse momentum component to one of the quantized values within the leads. This process results in collimation [47] of the incoming electrons, which are injected into the dot in a highly directed beam (figure 12). In sufficiently small dots, the discrete nature of the electronic energy spectrum may also remain resolved, in spite of the coupling that is provided to the external environment [79, 80]. With the collimation generated by the input contact, and the discrete nature of the dot states, Akis *et al* have argued that transport through the dots should be dominated by a small number of orbits that are selected by the point contact leads [75]. The selection process itself is thought to arise in a manner in which those orbits within the dot whose momentum components closely match those of the incoming electron beam are preferentially excited. The stability of the selected orbits, inferred from experiment, is thought to be a consequence of the collimation provided by the input point contact and the discrete quantization within the dot [70]. Equivalently, this process may be viewed from a quantum mechanical rather than a semi-classical perspective, as has recently been emphasized by Zozoulenko *et al* [77, 78]. These authors considered how electron transport through open dots may be described as a mode-matching process, in which quantum mechanical states within the dot couple selectively to one-dimensional modes in the point contact leads. Their approach involves expressing the wavefunction (Ψ) of an open square dot as a selective superposition of eigenstates of its *closed* counterpart (figure 13):

$$\Psi(x, y, E) = \frac{2}{L} \sum_m \sum_n c_{mn}(E) \sin \frac{m\pi x}{L} \sin \frac{n\pi y}{L} \qquad (5)$$

where L is the size of the dot and the c_{mn} are suitable expansion coefficients. Using equation (5), Zozoulenko *et al* were able to show that Ψ typically consists of a superposition of a *small*

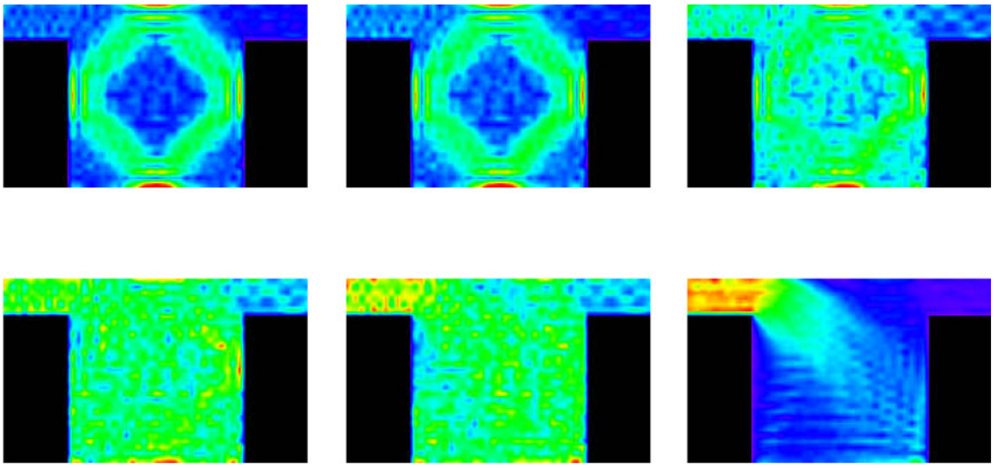


Figure 14. Calculated probability density in a 0.3 μm quantum dot at a magnetic field of 0.28 T. Lighter regions correspond to enhanced probability density and calculations are performed for different values of the phase breaking time. From top left to bottom right, $\tau_\phi = \infty$, 500 ps, 100 ps, 50 ps, 10 ps and 1 ps, respectively. Viewed in reverse sequence, this figure may therefore be considered as illustrating the time-dependent growth of the wavefunction scar within the quantum dot. Figure reproduced with permission from [81].

number of closed dot states, whose momentum components closely match the quantized values within the leads [77]. This selective behaviour was even found to persist in cases where the dot leads were configured to support as many as ten modes.

At temperatures where electron phase coherence is maintained over long distances, interference of the selectively excited dot states may give rise to wavefunction *scarring*, according to which the probability density in the dots becomes organized along the path of a small number of semi-classical orbits [69, 75, 76]. An example of the scarring is shown in figure 14, in which the wavefunction clearly shows the imprint of a diamond-shaped orbit. Using numerical simulations, Akis *et al* have shown that the details of the scarring are not independent of magnetic field but instead *recur* periodically as the field is varied [75]. These authors also found that the basic field scale for recurrence of the scars closely corresponds to the fundamental periodicities of the conductance fluctuations measured in experiment. The scarring is thought to be built up in a highly recursive process in which electrons undergo multiple traversals of the same basic orbits while maintaining phase coherence [70, 71, 81]. An important requirement for observation of the scarring is therefore that electrons remain coherently trapped in the dot for long time periods. In figure 14, for example, the diamond scar is only resolved for phase-breaking times in excess of 100 ps, by which time the electron has undergone roughly a hundred traversals of the dot. This property of the scarring results in a strong sensitivity to temperature variations, consistent with the results of experiment, which reveals the fluctuations to be quenched by a temperature of a few degrees kelvin [70]. It therefore seems worth re-emphasizing that both the periodic fluctuations in the conductance, and the associated wavefunction scarring, are associated with the interference of *long* semi-classical orbits.

While the regular nature of the magneto-conductance fluctuations in open dots was apparent from the earliest experimental studies, little consideration was given to their detailed origin [16, 22]. The connection between the fluctuations and the discrete density of states of open dots was first emphasized by Persson *et al* who studied electron transport in circular dots [82, 83]. More recently, Christensson *et al* have used temperature dependent studies

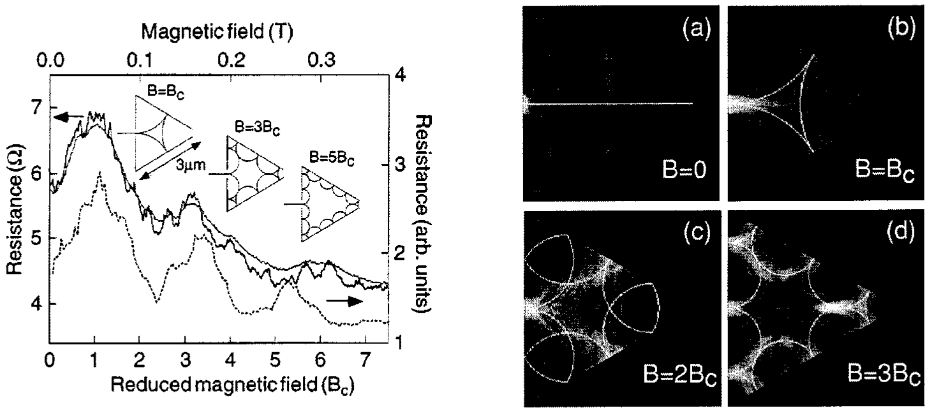


Figure 15. Classical and quantum features in the magneto-resistance of a triangular quantum dot. The left hand figure shows the measured magneto-resistance at 4.4 K (thin solid curve) and 0.3 K (thick solid curve). Indicated are the trajectories of electrons that are thought to be related to the local maxima of the resistance at $B/B_c = 1$, 3 and 5. The dashed line shows the results of numerical simulations. In the right hand figure, classical simulations of electron transport in the dots are presented. Superpositions of the trajectories of 5000 electrons injected through the side opening at the magnetic fields indicated are shown. Note that only the trajectories of electrons *reflected* by the billiard are plotted. At $B/B_c = 1$ and $B/B_c = 3$ the probability of reflection is high and the magneto-resistance reaches a maximum. The reflecting trajectory at $B/B_c = 2$ is unstable, however, and few electrons follow this or similar trajectories. Consequently, no magneto-resistance maximum is observed at this magnetic field. Figure reproduced with permission from [84]. Copyright 1998 by the American Physical Society.

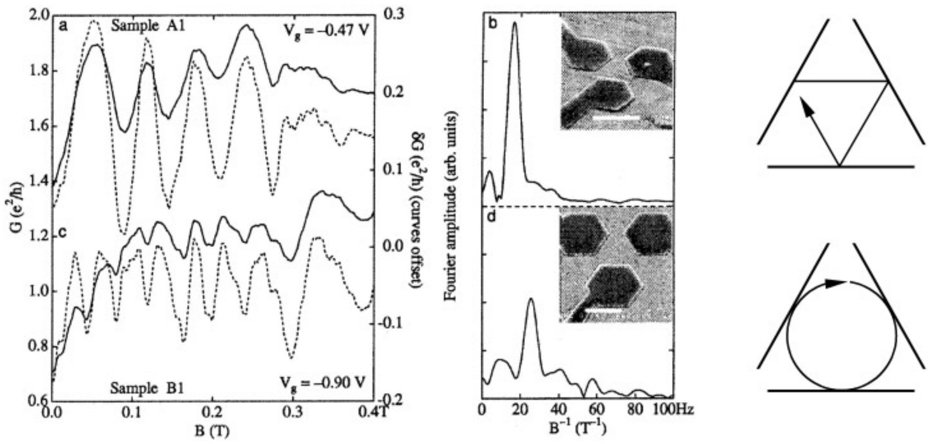


Figure 16. The low-temperature (20 mK) magneto-conductance of three-terminal triangular dots shows highly regular oscillations that are considerably stronger than any random fluctuations. The oscillations are shown in the left hand figure while their Fourier spectra are presented in the right hand plot. A single peak dominates the spectrum, suggesting that just a single orbit dominates transport. The orbit takes a triangular form at weak fields but gradually evolves into a cyclotron orbit at higher fields, as illustrated in the figure. The dot geometries studied are shown as insets in which the white spacer bars indicate a length of 1 μm. Figure reproduced with permission from [85]. Copyright 1998 by the American Physical Society.

to link specific classical electron trajectories to the quantum mechanical fluctuations in triangular-shaped dots [84]. At temperatures of a few degrees kelvin, quantum interference

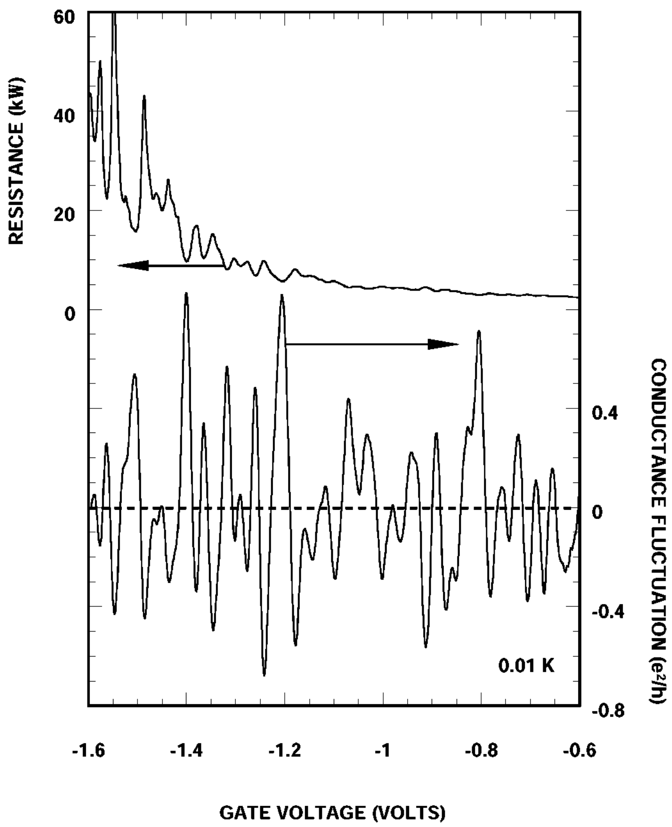


Figure 17. The low-temperature (0.01 K) conductance of split-gate dots shows a series of highly regular oscillations when their gate voltage is varied. The results shown here were obtained for the same dot geometry as that shown in figure 11. The upper curve shows the measured variation of resistance with gate voltage while the lower conductance oscillations were obtained after subtracting a monotonic background from the raw data. Unpublished data.

effects were found to be suppressed and the magneto-resistance instead exhibited a series of peaks related to the formation of magnetically focused orbits in the dots (figure 15). At millikelvin temperatures, however, the magneto-resistance showed periodic oscillations which were argued to result from an Aharonov–Bohm effect involving a particular focused orbit, a conclusion that was supported by the results of classical and quantum simulations (figure 15). This particular orbit was found to be selected by its short length and relatively high stability, while other more unstable orbits were not found to be important for classical or quantum transport through the dot. In another report, Bøggild *et al* studied electron transport in triangular-shaped dots with a novel three-lead geometry [85]. High-temperature features in the magneto-resistance of these dots were ascribed to classical focusing effects, in which electrons are directed into different leads at certain magnetic fields. Periodic magneto-conductance oscillations were observed at millikelvin temperatures, however, and were attributed to interference involving a dominant semi-classical orbit. This orbit was found to be robust to changes in both magnetic field and gate voltage, consistent with the earlier results of Bird *et al* [69]. The orbit was also argued to have a triangular form at weak fields, which gradually evolved to form a cyclotron orbit with increasing magnetic field (figure 16). A similar transition has also been studied by Zozoulenko *et al* [73].

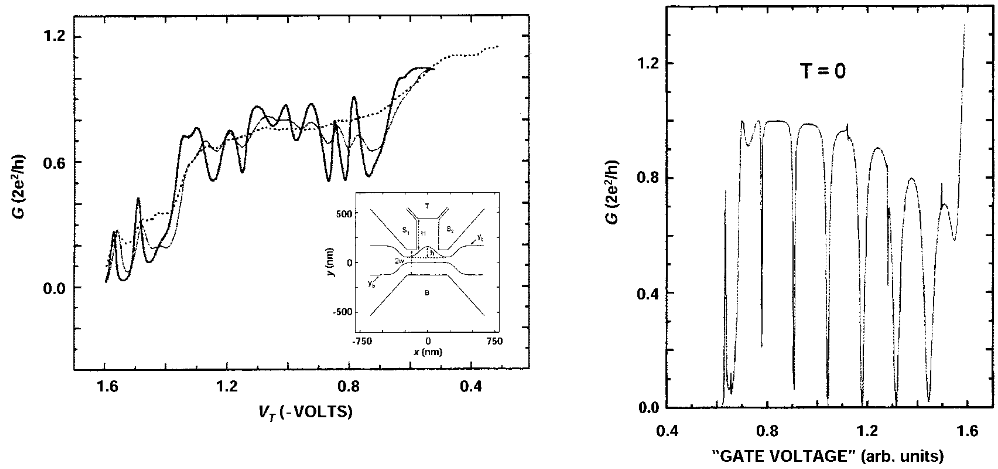


Figure 18. The left hand figure shows measured conductance traces of an electron stub tuner at three different temperatures: 0.09 K (thick solid line), 1 K (thin line) and 2.5 K (dotted line). The inset shows the basic gate geometry of the electron stub tuner, which is defined by four surface gates. The stub has lithographic height $H = 0.3\,\mu$m and width $2w = 0.25\,\mu$m. The quantum point contact leads have lithographic widths of 0.25 μm and the effective geometry of the tuner under gate bias is demarcated by the lines y_t and y_b. The right hand figure shows the computed variation of the conductance of the stub tuner at zero temperature. Figure reproduced with permission from [90]. Copyright 1999 by the American Institute of Physics.

Regular conductance oscillations, with similar characteristics to those discussed above, have also been observed in experiments in which a suitable gate voltage is varied at *fixed* magnetic field [82, 86–90] (figure 17). In the early study by Hirayama and Sadu it was suggested that the oscillations reflect the gate-voltage-induced movement of individual dot states past the Fermi surface [87]. Other authors have argued, however, that, rather than providing a spectroscopy of individual dot states, the oscillations instead reflect the details of a more robust shell structure that is formed at highly degenerate points in the dot spectrum [82, 91]. More recently, Bird *et al* have argued that the oscillations are related to the periodic recurrence of certain wavefunction scars with gate voltage, reminiscent of the behaviour found in magneto-transport studies [92, 93]. The latter authors analysed the properties of the oscillations in dots with different lead alignments and found their results to be consistent with different scars being excited by the distinct lead geometries [93]. Debray *et al* have focused on the behaviour exhibited by T-shaped *electron stub tuners*, which are essentially strongly confined quantum dots, and also report periodic oscillations in the conductance as gate voltage is varied [90]. A theoretical analysis of their results suggests that the oscillations are related to reflection resonances of electron waves from the resonant states within the stub (figure 18). This conclusion is supported by the later work of Bird *et al* who also found the oscillations to be related to the movement of *specific* dot states past the Fermi surface with increasing gate voltage [93].

6. Zero-field resistance peak in open dots

Weak localization is a well known correction to the conductance of disordered systems and results from a process known as coherent backscattering [94], in which diffusing electrons return to their origin after scattering within a disordered medium. At sufficiently low temperatures, constructive interference of the backscattered electrons and their time-reversed counterparts yields an enhancement of the sample resistance that is suppressed by the application of a magnetic field. Consequently, the magneto-resistance is found to exhibit a peak near zero magnetic field, whose amplitude depends on the details of electron scattering in the disordered medium. A similar resistance peak is also often exhibited by open quantum dots and has been argued to result from the ballistic analogue of weak localization, in which electrons backscatter within the dot through a series of collisions with its confining walls. According to one semi-classical theory, the lineshape of this peak is expected to provide a probe of the electron dynamics in the dot, with Lorentzian and linear magnetic field dependences predicted for chaotic and regular scattering, respectively [21, 22].

In order to clearly resolve a zero-field peak in experiment, it is necessary to suppress the influence of the surrounding fluctuations, which may dominate the low-temperature magneto-resistance. This may be achieved in a number of different ways, among which include measuring the response of multiply connected arrays of dots [22]. This approach was employed by Chang *et al* [22], who measured the magneto-resistance of circular and stadium-shaped dots and found lineshapes consistent with the predictions of [21]. In another approach, magneto-resistance traces are measured at a number of different gate voltages and are averaged to give a well resolved peak [18, 95]. This approach was recently used to study the temperature dependence of the phase-breaking time in open dots [32] and was found to give results consistent with other experimental approaches. Another technique for obtaining a clear peak involves increasing the measurement temperature to a few degrees kelvin, which is found to quench the fluctuations while leaving the central peak resolved [96, 97]. In studies of an array of nominally regular dots, the peak lineshape was found to evolve from linear to Lorentzian forms when such a temperature increase was performed and this was argued to result from a thermally induced onset of chaos at the higher temperatures [22]. In other studies of nominally regular dots, a gate-voltage-induced transition from a linear to a Lorentzian peak was found to occur as the dot leads were opened to support several modes [64, 96] (figure 19). This behaviour was speculated to result from an onset of chaos in more open dots and a subsequent theoretical study proposed that the relevant driving mechanism is an associated modulation of profile-rounding at the dot corners [97].

While the lineshape of the zero-field peak obtained in several experiments appears to provide good agreement with theoretical predictions, a recent report has suggested that the peak may *not* be a good indicator of chaos in open dots [98]. The motivation for this report was provided by the results of a number of more recent experiments, whose results appear to contradict the predictions of [21]. Lee *et al*, for example, studied the zero-field peak in a circular dot and found this to exhibit a Lorentzian lineshape [33]. A similar result was reported by Berry *et al* who also studied the magneto-resistance of circular-shaped dots [99]. Even more surprisingly, the low-temperature magneto-resistance of pairs of quantum point contacts has been found to exhibit a Lorentzian peak, in spite of the fact that there is no mesoscopic dot present to provide chaotic scattering in these structures [100, 101]. Motivated by the notion that magneto-transport studies of open dots provide a selective spectroscopy of their density of states (see section 4 above), Akis *et al* have suggested that the observation of a zero-field peak results from energy averaging of this discrete spectrum [98]. The basic idea is that at the finite temperatures where experiment is performed transport measurements of the dots sample a finite window in the density of states. The width of this window in turn depends on temperature, the degree of dephasing and the strength of the coupling that exists between the dot and its external environment. By averaging magneto-resistance traces computed at different energies, Akis *et al* have found that both Lorentzian and linear lineshapes may be observed in the same dot geometry! The generic nature of this result has been confirmed in studies of both hard-walled and realistically computed quantum dot profiles [98].

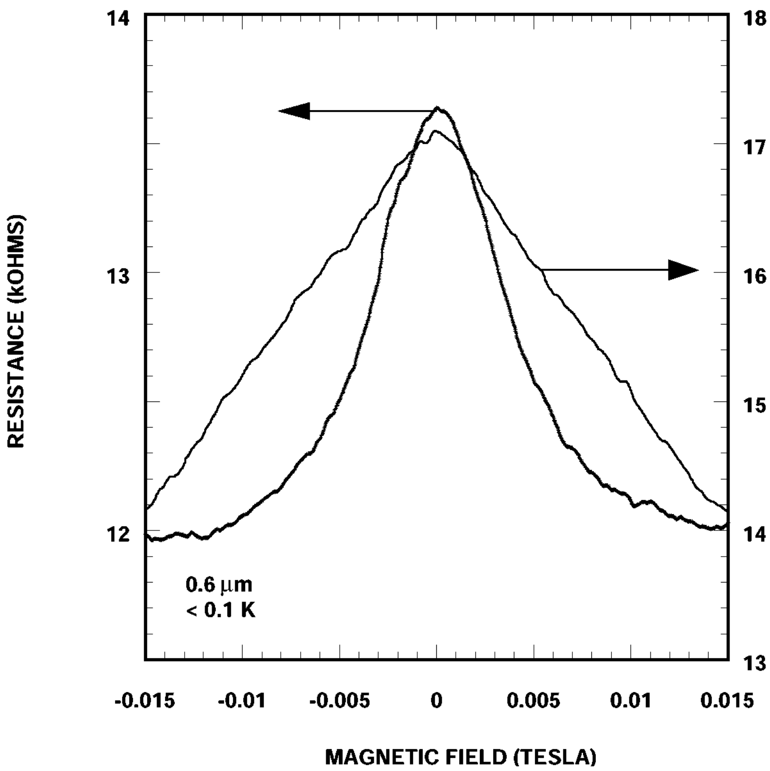


Figure 19. The low-temperature magneto-resistance (<0.1 K) measured in a 0.6 μm dot shows a transition from a Lorentzian to a linear zero-field peak lineshape when gate voltage is varied. The Lorentzian peak is plotted with symbols (+) and the linear peak is obtained for a more negative voltage applied to the split gates of the dot (note the change in zero-field resistance for the two dots).

7. Discussion: outstanding issues and future work

While a number of approaches have been employed to study electron dephasing in open dots [32, 35, 36, 38], a partially consistent picture now appears to be emerging from the results of these studies. In most experiments, the phase-breaking time is found to vary roughly inversely with temperature, consistent with the results of a recent theory that assumes dephasing to arise from the Coulomb interaction of electrons in the dots [40]. An unaccounted feature of experiment is the low-temperature saturation of the phase-breaking time, observed in some studies. This has been independently ascribed to a breakdown of the semi-classical model used to extract dephasing times [35] and a change in the dimensionality of the phase-breaking process at low temperatures [36]. It is well known from studies of disordered systems that the phase-breaking time in one-dimensional quantum wires may exhibit a similar saturation to that discussed here [42, 43, 102–104]. This has recently been argued to result from dephasing that is induced by the zero-point motion of electrons in the wire [104]. This interpretation remains the subject of controversy [105, 106], however, and the connection between the saturation observed in wires and dots is at present unclear. Other unexplained features of experiment include the absence of a well defined scaling of the phase-breaking time with dot size [32, 38] and a strong sensitivity to dot-to-dot variations [38]. In this regard, it seems clear that a theory of dephasing that accounts for the influence of remnant disorder in the dots is required. As for future experiments in this area, a relatively unexplored avenue for the study of dephasing is provided by non-equilibrium transport studies [49, 51, 52], which should provide a powerful tool for distinguishing between different relaxation processes in the dots.

In studies of the fractal nature of the magneto-conductance fluctuations in open dots, by far the most controversial issue concerns the origin of the exactly self-similar features reported in a Sinai billiard [64]. During the final stages of preparation of this review, a pre-print was received in which a simple physical model was used to reproduce the self-similarity seen in experiment [107]. By combining oscillatory contributions to the conductance, with flux periods of both h/e and $h/2e$, these authors were able to generate self-similar magneto-resistance structure on four magnetic field scales. Crucially, these authors found that close agreement between experiment and theory was only possible if the electrons orbits contributing to the magneto-conductance were assumed to follow a non-trivial distribution. The role played by the device structure in establishing this distribution, however, remains undetermined at present.

Other studies of quantum dots reveal their magneto-conductance to be dominated by a small number of stable orbits that are selected through coupling conditions with the point contact leads [70, 77, 82, 84, 85]. At temperatures where phase coherence is maintained over long distances, interference of the selectively excited orbits gives rise to wavefunction scarring that recurs periodically with gate voltage or magnetic field. The influence of the scarring on the transport behaviour remains a source of controversy [108, 109], and some authors have disputed the significance of the scarred states for electron transport [109]. Crucially, though, the scarring implies a non-uniform sampling of phase space in the dots, which is inconsistent with the presence of chaos. Instead, it has been suggested that electron transport in open dots is quite regular in nature, being dominated by a few orbits that are stabilized by the collimating action of the point contact leads and the discrete quantization within the cavity itself [71]. These notions appear consistent with the recent suggestion that the zero-field peak seen in the magneto-resistance of the dots may not be used as a reliable indicator of chaos [98]. Since the influence of specific orbits is found to be more pronounced in dots of smaller size, in which the discrete quantization of the level spectrum is more clearly resolved, an important implication is that the properties of these dots should no longer be well described by random matrix theory, which derives from an assumption of ergodic scattering in the dots.

A surprising aspect of the studies discussed above is the manner in which single-particle models of electron transport may be used to successfully account for much of their observed behaviour. Recently, however, a number of experiments have focused on the behaviour exhibited by *isolated* dots, in which electron–electron interaction effects are expected to play a far more important role [24–26, 110–116]. While a thorough discussion of these studies goes beyond the scope of this present review, significant deviations from the predictions of non-interacting random matrix theories have been reported [110, 111], pointing to the increased role that interaction effects play in closed dots. Of particular relevance to the studies presented here is recent evidence for the persistence of such charging effects in *open* dots [113, 115, 117, 118]. An important direction for future work will therefore be to clarify the relative roles of single-particle and many-body effects in open dots.

8. Summary

In this review, we have seen how quantum dots may be used to investigate signatures of classical scattering dynamics in the properties of quantum mechanical systems. At temperatures of the order of a degree kelvin and below, interference of coherent electrons is an important process in determining the electrical properties of the dots and the details of this interference may be probed using transport studies. Magneto-transport experiments are of particular value here and provide information on the electron dephasing time. In addition to the existence of fractal magneto-conductance fluctuations, these studies also reveal the presence of selectively excited orbits in the dots. A crucial role in selecting these orbits is thought to be played by the quantum mechanical lead openings which, when configured to support a small number of modes, inject electrons into the dot in a highly collimated beam. With the coupling provided by these few mode leads, opening the dot to its external environment does not obscure its discrete spectrum, as has been suggested previously. Rather, the different states of the dot are broadened non-uniformly and transport measurements may be used to provide a *selective* spectroscopy of this *filtered* density of states.

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